FROM NEWTON TO CHAOS

by Robert May

The vision given to us by Newton and by those who followed in the age of Enlightenment is of an orderly and predictable world, governed by laws and rules - laws and rules which can best be expressed in mathematical form. If the circumstances are simple enough (for instance, a planet moving around a sun, bound by the inverse square law of gravitational attraction), then the system behaves in a simple and predictable way. Effectively unpredictable situations (for instance, a roulette ball whose fate - the winning number - governed by a complex concatenation of croupier's hand, spinning wheel, and so on) were thought to arise only because the rules were many and complicated.

Over the past 20 years or so, this Newtonian vision has splintered and blurred. It is now widely recognised that the simplest rules or algorithms or mathematical equations, containing no random elements whatsoever, can generate behaviour which is as complicated as anything we can imagine.

This mathematics which 'is different' is the mathematics of 'deterministic chaos'. What it says is that a situation can be both deterministic *and* unpredictable; that is, unpredictable without being random (on the one hand) or (on the other hand) attributable to very complicated causes.

'Simple' as they may be in themselves, these chaos-generating equations have the property of being 'nonlinear'. In a 'linear' equation you can 'guess ahead'. Imagine a road lined with telegraph poles in a perspective drawing. Given two or three poles, you can easily draw in the rest for yourself. But nature often draws itself differently, using nonlinear equations. Imagine a river running alongside the road. The water has flat bits and bumpy bits. But however many I draw in for you, there is no way for you to tell (with a real river) where the next flat bit or bumpy bit is going to be. This holds true on every scale. Look down from a balloon and you'll see that parts of the bumpy bits look relatively flat. Put your face close to the water and you'll see that the flat bits contain relatively bumpy bits. The maths is the same for each case, and equally unpredictable.

In this sense, nonlinear means two and two do not necessarily make four. Much of physics and other areas of science where so much progress has come are linear (or at least decomposable into essentially linear bits). And so mathematical texts and courses have focused on linear problems. But increasingly it seems that most of what is interesting in the natural world, and especially in the biological world of living things, involves nonlinear mathematics. It was biologists - working on the ups and downs of animal population - who were among the first to see that not only can simple rules give rise to behaviour which looks very complicated, but the behaviour can be so sensitive to the starting conditions as to make long-term prediction impossible (even when you know the rule).

There is a flip side to the chaos coin. Previously, if we saw complicated, irregular or fluctuating behaviour - weather patterns, marginal rates of Treasury bonds, colour patterns of animals or shapes of leaves - we assumed the underlying causes were complicated. Now we realise that extraordinarily complex behaviour can be generated by the simplest of rules. It seems likely to me that much complexity and apparent irregularity seen in nature, from the development and behaviour of individual creatures to the structure of ecosystems, derives from simple - but chaotic - rules. (But, of course, a lot of what we see around us is very complicated because it is intrinsically so!)

I believe all this adds up to one of the real revolutions in the way we think about the world. Knowing the simple rule or equation that governs a system is not always sufficient to predict its behaviour. And, conversely, exceedingly complicated patterns or behaviour may derive not from exceedingly complex causes, but from the chaotic workings of some very simple algorithm. Ultimately, the mathematics of chaos offers new and deep insights into the structure of the world around us, and at the same time raises old questions about *why* abstract mathematics should be so unreasonably effective in describing this world. ■

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CHLOË: THE FUTURE IS ALL PROGRAMMED LIKE A COMPUTER. THAT'S A PROPER THEORY, ISN'T IT?

VALENTINE: YES. THERE WAS SOMEONE, FORGET HIS NAME, 1820s, WHO POINTED OUT THAT FROM NEWTON'S LAWS YOU COULD PREDICT EVERYTHING TO COME - I MEAN, YOU'D NEED A COMPUTER AS BIG AS THE UNIVERSE BUT THE FORMULA WOULD EXIST.

CHLOË: BUT IT DOESN'T WORK, DOES IT?

VALENTINE: NO. IT TURNS OUT THE MATHS IS DIFFERENT.

VALENTINE: IT MAKES ME SO HAPPY. TO BE AT THE BEGINNING AGAIN, KNOWING ALMOST NOTHING...A DOOR LIKE THIS HAS CRACKED OPEN FIVE OR SIX TIMES SINCE WE GOT UP ON OUR HIND LEGS. IT'S THE BEST POSSIBLE TIME TO BE ALIVE, WHEN ALMOST EVERYTHING YOU KNEW IS WRONG.